Optimal Monetary Policy with Asymmetric Shocks and Rational Inattention*

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Abstract

This paper investigates the optimal monetary policy in a two-sector model with imperfect common knowledge between firms due to limited attention. A welfare criterion based on the utility of consumers is derived to evaluate the real effects of monetary policy. When sectoral productivity shocks are symmetric, complete price stabilization is optimal. However, with asymmetric sectoral productivity shocks, price stabilization is no longer optimal. When responding to these shocks, central banks face tradeoffs between output and price stabilization. The optimal monetary policy places more weight on price stabilization when firms have more information processing capacity.

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1 Introduction

Consider an economy with two sectors. Firms with limited attention face sector-specific shocks. We ask the following questions. How do firms allocate their limited attention to these shocks? How do they make their price-setting decisions? What is the optimal monetary policy? The purpose of this paper is to construct a model to answer these questions.

In this paper we explore the implications of imperfect common knowledge and asymmetric shocks in a two-sector model for the optimal monetary policy. Our formulation of incomplete information is based on the concept of 'rational inattention' proposed by Sims (1998, 2003, 2010), which can also be viewed as modeling 'optimal learning' when agents learn with error due to information processing constraints. It is assumed that all information about the exogenous shocks is available to agents in the economy, but there is a limit on their ability to process information. By choosing their information structure optimally in response to the changing environments in the economy, they may acquire different pieces of information endogenously. In our model firms continuously observe noisy signals about theses shocks. Their learning choice is how much attention to allocate to each shock. We adopt this framework to study the mechanism of price adjustment by firms in two different sectors.

Firms' price adjustment has long been a central issue for the explanation of monetary non-neutrality. The modeling mainly falls into several classes: for example, staggered price; see Calvo (1983), fixed adjustment costs; see Barro (1972) and Caplin and Spulber (1987), sticky information; see Mankiw and Reis (2002) and rational inattention; see Sims (2003); Woodford (2003a) and Mackowiak and Wiederholt (2009). No matter what the mechanism is, the idea that complete price stabilization is optimal in response to shocks that cause efficient fluctuations of natural output is unanimously accepted. This paper adopts the rational inattention concept but reaches a more complete characterization of the optimal monetary policy. Specifically we show that with asymmetric sectoral productivity shocks, price stabilization is no longer optimal.

We develop a two-sector model with flexible prices and monopolistic competitive firms characterized by information processing constraints. There are sector-specific productivity shocks. In response to these shocks, firms optimally allocate their limited information capacity and make decisions on price adjustment. Due to limited attention, firms' optimal prices equal to the optimal price under perfect information with an observational noise. The observational noise determines the price dispersion in that sector. With rational inattention, the variance of the noise is proportional to the variance of the optimal price under perfect information, which is endogenously determined by the monetary policy and the underlying shocks. Monetary policy hence influences price dispersion in both sectors.

The other important feature of our framework is the existence of asymmetric shocks across sectors. In a recent paper, Mackowiak and Wiederholt (2009) introduces idiosyncratic shocks to firms. In their model, these idiosyncratic shocks are firm-specific. Firms will pay no attention to the shocks of other firms. When aggregating prices, these disturbances cancel out and have no direct effect on the aggregate variables. In our model, shocks are sector-specific. Firms are not excluded from observing sectoral shocks and the asymmetry of shocks has major implications for firms' price setting. We demonstrate in this paper that firms pay more attention to shocks in their own sector and less to those in the other sector. As a result, the impact of the sectoral shocks on firms' price setting shows a pattern of asymmetry. When considering optimal monetary policy, this asymmetric property plays an important role and leads to conclusions different from the current literature.

Main Results The equivalence between the output and price stabilization holds under the two informationally extreme situations and the symmetric case. When information is perfect, the economy responds to productivity shocks efficiently and the monetary neutrality holds. It is optimal for the central bank to keep the price level at its natural rate. When firms have no information processing capacity, monetary policy has real effects only. Central bank should also stabilize the output gap. When shocks are symmetric, it is the case discussed in Adam (2007). In order to minimize the price dispersion, central banks should stabilize the price level so that firms pay no attention to the disturbances, which replicates the no information case and the output gap is also optimal.

In our model, with asymmetric sectoral productivity shocks, price stabilization is no longer optimal. When responding to these shocks, the central bank face tradeoffs between output and price stabilization. The central bank should put some weight to the asymmetry of these sectoral shocks. If firms have no information processing capacity, central bank should stabilize the output gap. As information processing capacity becomes larger, firms will pay more attention to shocks in other sectors. Therefore, central bank should put more weight on the asymmetry.

The intuition for this result is the following. When shocks are asymmetric, shocks in different sectors influence firm's real marginal cost disproportionally. Shocks in the sector to which the firms belong have greater impact on firms' marginal cost in that sector. With information processing capacity, firms pay more attention to productivity shocks from the sector they are located in. The price dispersion is thus proportional to firms' observational noise, while, with rational inattention, the observational noise is proportional to the optimal price setting under perfect information. Hence, the price dispersion in each sector is influenced by the asymmetric shocks as well as the monetary policy. It is also derived that the welfare relies on the output gap, the price dispersion in each sector and the deviation of the relative price between the two sectors from its natural rate. Due to the asymmetry of shocks, the central bank cannot stabilize the output gap and minimize the price dispersion at the same time. Apart from responding to fluctuations of the natural rate of output, the central bank should also put some weights to the asymmetry of these shocks to coordinate conflictive welfare objectives. Specifically, as firms' information capacity grows, central bank should put more weight on price stabilization.

Related Literature This paper relates to two strands of literature. First, this paper relates to the large literature on imperfect information proposed by Phelps (1970) and Lucas (1973). This paper is inspired by Sims (2003, 2010), whose seminal work initiates the literature of rational inattention; see also, for example, Luo (2008), Woodford (2009) and Yang (2011). In this paper, 'learning' is different from that in the literature on learning. Agents in the learning literature make forecasts using observed data and update their forecast rules over time in response to errors. With rational inattention, agents update their information structure optimally by observing the underlying state of the economy imperfectly over time. As pointed in Sims (2003), agents have limited attention and 'learn' slowly about the state of the economy because they cannot process information unboundedly.

The main idea of rational inattention is that agents choose their information structure optimally. Since agents acquire their information intentionally, this choice is considered as a form of 'active learning' (see Veldkamp (2011) p.4). Woodford (2003a) considers imperfect common knowledge induces by limited attention and finds that the model fits well with the evidence found in VAR literature. This paper contributes to this literature by discussion the optimal monetary policy in a more realistic setting with asymmetric sectoral shocks.

Secondly, this paper is related to the literature on optimal monetary policy. Sargent and Wallace (1975) studies the optimal policy with the assumption of Lucas (1973). Ball et al. (2005) concludes that complete price stabilization is optimal in response to aggregate technology shocks. Price setters in their paper update information with an exogenous probability and the information structure in their paper is exogenous. Adam

(2007) solves the maximization problem in a setting as the one in Woodford (2003a). Complete stabilization of the price level is still optimal with respect to productivity shocks. This paper can be viewed as an extension of Adam's work to the two-sector case. However, with asymmetric sectoral shocks, complete price stabilization is no longer desirable for the central bank.

Two close antecedents to our current paper are Aoki (2001) and Benigno (2004). They both construct a two-sector model to investigate the optimal monetary policy with sectoral asymmetry. Aoki (2001) constructs a two-sector economy where price is sticky in one sector and flexible in another. He finds that the optimal policy is to stabilize the price level in the sector with sticky price. Benigno (2004) shows that if degrees of nominal rigidities across regions are different, optimal monetary policy should give higher weight to the region with higher degree of nominal rigidity. Both paper show that central banks should pay attention to the asymmetry between sectors. Our paper complements their findings under rational inattention.

Over the last 10 years, there has also been rapid developments in research on micro data of firms' price setting.[1] More data have been collected and more details about price adjustment have been revealed.[2] For example, Maćkowiak et al. (2009) finds that firms tend to have quicker response to sector specific conditions compared with aggregate conditions. At the same time, it is necessary and practically useful to tackle the optimal policy problem under sectoral shocks. This work is a step toward this direction theoretically.

The rest of the paper is organized as follows. Section 2 presents a two-sector economy with monopolistic competition. Section 3 models firm's information processing. Section 4 derives the equilibrium of the economy. Section 5 analyses the optimal monetary policy. Section 6 concludes.

2 Model

In this section we construct a two-sector economy with flexible prices, a continuum of monopolistically competitive firms on $[0,1]$ and a central bank. We have $j, j = 1,2$ to denote each sector respectively. The firms on $[0, n_1)$ belong to sector 1 and and those on $[n_1, 1]$ belong to sector 2, where $n_2 = 1 - n_1$. In this paper, n_j can be viewed as the economy size of sector j. In the monetary union interpretation, n_j can be the economy

[1] One of the most important datasets are provided by the U.S. Bureau of Labor Statistics (BLS). Scanner data have also drawn a lot of attention recently.

[2] See Nakamura and Steinsson (2013) for a detailed discussion.

size of country j. Each sector faces idiosyncratic productivity shocks. Firms pay limited attention to these random shocks and choose their optimal prices. These assumptions imply that firms can not only observe shocks in the sector they locate in but also shocks in another sector, which is in accordance with the idea of rational inattention.

The central bank has perfect information about the shocks and controls the aggregate nominal spending. The bank commits to a policy rule to maximize the expected utility of a representative household.

For simplicity, we only consider an economy with temporary shocks, which means shocks are independent across periods. Therefore, it is equivalent to consider a static Ramsey problem as to a dynamic one, so we omit the time index t. Though it is interesting and important to extend the model to dynamic settings, the focus of this paper is on asymmetry of shocks and its normative implications.

2.1 Household problem

There is a representative household whose utility function is $\Omega = U(C) - V(L)$, where $U(\cdot)$ represents the utility of consumption and $V(\cdot)$ is the disutility of labor. C is represented as an index of total consumption denoted by

$$C = (n_1^{\frac{1}{\eta}} C_1^{\frac{\eta-1}{\eta}} + n_2^{\frac{1}{\eta}} C_2^{\frac{\eta-1}{\eta}})^{\frac{\eta}{\eta-1}}, \tag{1}$$

where C_1 and C_2 are index of consumption across the continuum of differentiated goods produced by sector 1 and 2 respectively given by

$$C_j = [n_j^{-\frac{1}{\theta}} \int_{N_j} c(i)^{\frac{\theta-1}{\theta}} \, \mathrm{d}i]^{\frac{\theta}{\theta-1}}, \quad j = 1, 2. \tag{2}$$

We use θ as the elasticity of substitution across goods produced within a sector. We allow the elasticity of substation between the consumption C_1 and C_2 η to be different from that within sectors. Here we introduce the overall price index defined as

$$P = (n_1 P_1^{1-\eta} + n_2 P_2^{1-\eta})^{\frac{1}{1-\eta}}. \tag{3}$$

which is also the minimum cost of obtaining a unit of C . P is the Dixit-Stiglitz price index of the composite differentiated good in sector j given by

$$P_j = [n_j^{-1} \int_{N_j} p(i)^{1-\theta} \, \mathrm{d}i]^{\frac{1}{1-\theta}}. \tag{4}$$

The demands for composite differentiated good j, produced in sector j, are

$$C_j = n_j C(\frac{P_j}{P})^{-\eta}, \tag{5}$$

and the demands for differentiated good i in sector j are

$$c(i) = n_j^{-1} C_j(\frac{p(i)}{P_j})^{-\theta}. \tag{6}$$

We assume there is a single labor market, where the representative household supplies homogenous labor, which means labor is mobile across sectors and firms.[3] The representative consumer chooses his consumption C and labor supply L to perform the following maximization:

$$\begin{aligned} &\underset{U,V}{\text{Max}} \quad U(C) - V(L) \\ &\text{s.t.} \quad\ \ 0 = WL + \Pi - T - G - PC \end{aligned}$$

where W is the wage rate in labor market, Π the nominal profits from firms, T the nominal transfers from central bank and G is the lump sum tax collected by the government. From the first order condition we get the expression for the real wage:

$$\frac{W}{P} = \frac{V'(L)}{U'(C)}. \tag{7}$$

2.2 Firms and price setting

In equilibrium, production equals consumption, which is $Y = C$, $Y_j = C_j$ and $y(i) = c(i)$. Thus we use Y, Y_j and $y(i)$ to denote production as well as consumption for simplicity.

There are a continuum of monopolistic competitive firms indexed by $i \in [0,1]$. The fraction of firms in sector 1 and 2 is n_1 and n_2 respectively. Monopolistic competition implies that firms can affect demand by setting different prices. Firm i in sector j supplies good i with labor L_i employed in the labor market according to a technology of the form

$$y(i) = \frac{L_i}{A_j}. \tag{8}$$

[3]The common labor market assumption may render the assumption of strategic complementarity in this paper implausible, as pointed in Woodford (2003b) Chapter 3. However, as long as the coefficient of relative risk aversion is small enough in the steady state, we can still assume that strategic complementarity holds in the model; $\xi < 1$.

A_j affects the sector specific productivity in sector j, the log of which follows a Gaussian distribution $lnA_j \sim N(0, {\sigma_j}^2)$. This kind of shock is an example of a shock with the following property: the economy's response to the shock under perfect information is efficient.

The nominal profit of firm i is given by

$$\Pi[p(i), \cdot] = (1+\tau)p(i)y(i) - WA_j y(i).$$

Here τ is the production subsidy paid by the government. We assume that τ equals $\frac{1}{\theta-1}$ so as to completely offset the distortion caused by monopolistic competition. Firm i in sector j chooses $p(i)$ so as to maximize its expected nominal profit while taking nominal wage rate W, composite consumption Y, and aggregate price index P as given. Log linearizing the first order condition of firm i gives an expression for the profit-maximizing price:

$$\hat{p}^*(i) = E[\widehat{P} + \xi\widehat{Y} + \omega n_1 \widehat{A}_1 + \omega n_2 \widehat{A}_2 + \widehat{A}_j | s_i], \tag{9}$$

where $\omega = \overline{Y}\frac{V''(\overline{Y})}{V'(\overline{Y})}$, which represents the elasticity of marginal cost with respect to labor supply. Shocks from different sectors have asymmetric effects on firms' optimal prices, because they have disproportional effects on firms' real marginal cost. Productivity shock A_j imposes greater impact on firms' price setting in sector j. This property is important for understanding the effects of monetary policy.

Substituting the expression of the natural rate of aggregate output $\widehat{Y}^n$, which is the output under flexible prices and perfect information[4]:

$$\widehat{Y}^n = -\frac{1}{\xi}[(\omega n_1 \widehat{A}_1 + \omega n_2 \widehat{A}_2) + (n_1 \widehat{A}_1 + n_2 \widehat{A}_2)].$$

into (9), we get:

$$\hat{p}^*(i) = E[\widehat{P} + \xi(\widehat{Y} - \widehat{Y}^n) + n_{-j}(\widehat{A}_j - \widehat{A}_{-j}) | s_i]. \tag{10}$$

where s_i represents the signal firm i receives, based on which firms make their pricing decisions. The parameter ξ is an indicator of the degree of strategic complementarity between the price setting decisions of different firms, which can be expressed as:

$$\xi = -\frac{U''(\overline{Y}) - V''(\overline{Y})}{U'(\overline{Y})}\overline{Y}.$$

[4]Please see Appendix A1 for its derivation.

The assumption of strategic complementarity, which is $0 < \xi \leq 1$, implies that the coefficient of relative risk aversion of representative household in steady state is small enough. Letters with hat represent variables that are expressed as percentage deviations from steady state. Letters with bar represents variables in steady state.

2.3 Central bank

Our main focus is on the normative analysis of optimal monetary policy, so we are not restricted to a specific monetary instrument. However, to complete the model, this paper assumes that central bank commits to a policy rule that maximizes the expected utility of the representative household by directly controlling the nominal spending Q, which equals the aggregate nominal demand given by

$$Q = PY \tag{11}$$

Log linearizing (11) gives:

$$\widehat{Q} = \widehat{P} + \widehat{Y}. \tag{12}$$

The expression of the aggregate demand can be derived from a typical Cash-In-Advance model. Following Rotemberg and Woodford (1997) and Woodford (2003b), we can derive a log-quadratic approximation of the utility of the representative household in this particular economy, which gives Lemma 1.

Lemma 1 *In this two-sector economy, the utility of the representative household can be approximated by taylor expansion up to second order as:*

$$\begin{aligned}\Omega &= U(Y) - V(L) \\ &= -\tfrac{1}{2}\overline{Y}U' \left\{ \xi(\widehat{Y} - \widehat{Y}_n)^2 + \alpha \textstyle\sum_{j=1}^{2} n_j^2 \widehat{P}_j^{\,2} + \beta[(\widehat{P}_1 - \widehat{P}_2) - (\widehat{P}_1^n - \widehat{P}_2^n)]^2 \right\} \\ &\quad + t.i.p + o(2).\end{aligned} \tag{13}$$

where $\alpha = \frac{(1-k)\theta}{2k}$ and $\beta = n_1 n_2 \eta$. $\widehat{P}_j^n$ represents the natural rate of price level in sector j[5], which is the price level under perfect information. The variable k which is defined in Section 3summarizes the information processing capacity, and t.i.p denotes terms that are independent of policy, o(2) means up to second order.

Proof.

[5]See (29) and (30) for the expression.

See Appendix A2.

From this expression, it is easily seen that the utility of the representative household relies on the output gap, the price dispersion in each sector and the deviation of the relative price between the two sectors from its natural rate. This result is quite intuitive: since the economy's response to productivity shocks is efficient, deviation from the natural output is suboptimal; the price dispersion in each sector causes inefficient relative price distortions which reduce welfare. Monetary policy should also stabilize relative prices as if prices are flexible in which case resources are allocated efficiently. The central bank tries to minimize this objective function by appropriately choosing monetary policy. In later sections we will find that normally there are tradeoffs between these three objectives.

The assumption that central bank has perfect information is somewhat realistic, for it is widely accepted that central bank are more concerned about the overall state of the economy. They hire economists and statisticians to closely follow the underlying shocks of the economy.

3 Information processing

In global game literature, see Morris and Shin (2003); the information structure is exogenous, which is set by the economist i.e. the modeler. Instead, the basic idea of rational inattention is the endogeneity of information structure. Following Sims (2003), we assume that firms can observe all existing information but have limited ability to process those information. Firms' information processing capacity is summarized by a parameter $K \geq 0$. By appropriately allocating their attention, firms choose their information structure optimally.

In this section, we first introduce information theory to quantify information flow. Then we derive firms' loss function of log-quadratic form. In the end we solve the optimal information structure of each firm by explicitly specify the signal it receives.

3.1 Information processing capacity

Firms are constrained by the amount of information they can process. This is intuitive because, for example, even if all information is public, before making decisions, managers have to take efforts to read and analyze reports and memos, which takes time and mental attention to incorporate this information into their decisions.

Following Sims (2003, 2010), we use the concept of entropy to quantify the information flow. Entropy is the measure of uncertainty of a random variable, which can be derived under several reasonable axioms; see Jaynes (2003). The entropy $H(X)$ of a random vector X that has multivariate normal distribution with variance matrix Σ is given by

$$H(X) = \frac{1}{2}log_2[(2\Pi e)^T det\Sigma].$$

According to information theory, conditional entropy describes the conditional uncertainty. If X and Y have a multivariate normal distribution, we can express conditional entropy of X given Y as

$$H(X|Y) = \frac{1}{2}log_2[2\Pi e)^T det\Sigma_{X|Y}].$$

With entropy and conditional entropy, mutual information $I(X|Y)$ is defined as

$$I(X|Y) = H(X) - H(X|Y). \tag{14}$$

Mutual information represents the uncertainty reduction of a random vector X after observing the signal Y. Intuitively, we restrict the information flow a firm can process by introducing the constraint on mutual information. The information processing capacity of firm i can be expressed by

$$I(Z_i; s_i) \leq K, \tag{15}$$

where Z_i are the vector of random variables firm i wants to observe and s_i is the signal it chooses to receive.

3.2 Optimal information structure

We first derive the loss function of firm i due to suboptimal price with imperfect information. From (10) in Section 1 we know the optimal price of firm i given signal s_i is

$$\hat{p}^*(i) = E[\widehat{P} + \xi(\widehat{Y} - \widehat{Y}^n) + n_{-j}(\widehat{A}_j - \widehat{A}_{-j})|s_i],$$

while the optimal price under perfect information equals[6]

$$\hat{p}^\circ(i) = \widehat{P} + \xi(\widehat{Y} - \widehat{Y}^n) + n_{-j}(\widehat{A}_j - \widehat{A}_{-j}). \tag{16}$$

[6]Optimal price of firm i under perfect information is the price set by firm i if all the other firms in the economy has limited attention except firm i.

Here we have $\hat{p}^*(i) = E[\hat{p}^\circ(i)|s_i]$. The loss profit of firm i due to suboptimal price can be expressed by

$$\widehat{\Pi}[\hat{p}^\circ(i), \cdot] - \widehat{\Pi}[\hat{p}^*(i), \cdot] = \tfrac{\theta n_j \overline{Y}}{2\overline{P}}[\hat{p}^\circ(i) - \hat{p}^*(i)]^2. \tag{17}$$

Appendix A.3 gives detailed derivation of this equation.

The signal s_i chosen by firm i affects $p^*(i)$, and then will affect the loss profit. The optimal information structure of firm i is determined by appropriately selecting s_i to minimize its profit loss under the constraint of information processing capacity. We assume that the price function $\hat{p}^\circ(i)$ is linear with respect to $\widehat{A}_1$ and $\widehat{A}_2$,[7] which is given by $\hat{p}^\circ(i) = \gamma Z$, where Z is a vector of uncorrelated fundamental shocks. In this paper, $Z = (\widehat{A}_1, \widehat{A}_2)$, $Z \sim N(0, \Sigma)$, and $\Sigma = diag(\sigma_1^2, \sigma_2^2)$. Under this formulation, the constrained optimizing problem of firm i with information processing capacity is the same as that in Adam (2007), which is[8]

$$\begin{aligned} &\underset{c,\varepsilon_i}{\text{Max}} \quad -E[[\hat{p}^\circ(i) - \hat{p}^*(i)]^2|s_i] \\ &\text{s.t.} \\ &\qquad s_i = c'Z + \varepsilon_i \\ &\qquad H(Z) - H(Z|s_i) \leqslant K. \end{aligned} \tag{18}$$

In Sims (2003), the author has proved that in the normal linear-quadratic case, the optimal information structure is the random state variable with an i.i.d. error.[9] His result justifies the form of information structure here, which is $s_i = c'Z + \varepsilon_i$.

The solution to this maximizing problem is given by[10]

$$c' = \gamma, \tag{19}$$

$$\sigma_{\varepsilon_i}^2 = \frac{1}{2^{2K} - 1} var[p^\circ(i)]. \tag{20}$$

This result is intuitive. It implies that firms choose to observe a signal regarding the

[7]It is shown in Section 4 that the price function is linear.

[8]Some may wonder why the constraint is $H(Z) - H(Z|s_i) \leqslant K$ but not $H(\gamma Z) - H(\gamma Z|s_i) \leqslant K$. Actually, they are the same. See Mondria (2010) A.3 for a proof.

[9]Sims' setup is the optimal control problem between optimal behavior and the random state variable. But if we construct a one for one correspondence between signal and optimal behavior, the optimal control problem in this paper is the same as that in Sims (2003).

[10]The reader may need to refer to Section 4.1 and Appendix A.3 of Adam (2007) for detailed derivation. However, there is a difference between this paper and Adam (2007). In his paper, it is assumed that the central bank reveals information to firms to maximize their profits. Here we assume firms collect information directly.

optimal price under perfect information to maximize their profits. Any other kinds of information structure will waste their limited attention.[11] An important property of the optimal information structure is that the conduct of monetary policy will affect the accuracy of the signals firms receive. This is easily seen by expressing $\hat{p}^{\circ}(i)$ in terms of the nominal spending Q:

$$\hat{p}^{\circ}(i) = (1-\xi)\widehat{P} + \xi(\widehat{Q} - \widehat{Y}^{n}) + n_{-j}(\widehat{A}_{j} - \widehat{A}_{-j}). \tag{21}$$

This feature is important in later sections on analyzing optimal policy. With (19) and (20), the conditional expectation of optimal price, i.e. $\hat{p}^{*}(i)$ is given by

$$\begin{aligned} \hat{p}^{*}(i) &= ks = k[\hat{p}^{\circ}(i) + \varepsilon_i] \\ &= k[(1-\xi)\widehat{P} + \xi(\widehat{Q} - \widehat{Y}^{n}) + n_{-j}(\widehat{A}_{j} - \widehat{A}_{-j}) + \varepsilon_i], \end{aligned} \tag{22}$$

where k is the Kalman gain which equals:

$$\begin{aligned} k &= \frac{var[\hat{p}^{\circ}(i)]}{var[\hat{p}^{\circ}(i)] + \sigma^2_{\varepsilon_i}} \\ &= 1 - 2^{-2K}. \end{aligned}$$

The kalman gain k is an alternative variable to measure the information processing capacity of firms. When there is no capacity, $K = 0$ and $k = 0$. $k = 1$ represents infinite capacity $K = \infty$.

Integrating (22) in sector j we get the price function in that sector:

$$\widehat{P}_{j} = k[(1-\xi)\widehat{P} + \xi(\widehat{Q} - \widehat{Y}^{n}) + n_{-j}(\widehat{A}_{j} - \widehat{A}_{-j})]. \tag{23}$$

where the idiosyncratic observing noises cancel out.

4 Equilibrium

Due to limited processing capacity, firms face idiosyncratic noise, so the common knowledge of the state of the economy between firms breaks down. Before making decisions, each firm has to infer what other firms believe, which depends on what

[11] Please see Appendix A.4 for the proof. Mondria (2010) also proves that if the objective function is exponential, a linear signal is also optimal than observing independent signals; see Mondria (2010) Lemma 2.

others believe about others, and so on ad infinitum. There are two ways to tackle this problem. One is explicitly displaying higher order beliefs of firms. For example, see Woodford (2003b), Morris and Shin (2002). The other one makes use of the method of rational expectation equilibrium by guessing the price function; see Paciello and Wiederholt (2011). This paper solves equilibrium following the latter one for simplicity and tractibility.

Given monetary policy, an equilibrium of the model is defined as a set of signals, $\{s_i\}$, output of each sector, $\{Y_j\}$, the aggregate output, $\{Y\}$, and the price functions, $\{P_j\}$, $\{P\}$ such that:

(i) Given $\{Y_j\}$, $\{Y\}$, $\{P_j\}$ and $\{P\}$, firm i chooses the signal $\{s_i\}$ optimally so as to maximize its expected profit. After receiving signal, it sets its price according to (9).

(ii) The sectoral price level $\{P_j\}$ is given by (4), and the aggregate price level $\{P\}$ is given by (3). The aggregate output $\{Y\}$ is given by (11), and the sectoral output $\{Y_j\}$ is given by (5).

Proposition 1 (Firms' price-setting) *If (21) and (23) holds, the following price functions of each sector are the solutions to the equilibrium defined above.*

$$\widehat{P}_1 = \frac{k}{1-k(1-\xi)}\{\xi(\widehat{Q}-\widehat{Y}^n) + n_2[1-k(1-\xi)](\widehat{A}_1-\widehat{A}_2)\}, \tag{24}$$

$$\widehat{P}_2 = \frac{k}{1-k(1-\xi)}\{\xi(\widehat{Q}-\widehat{Y}^n) + n_1[1-k(1-\xi)](\widehat{A}_2-\widehat{A}_1)\}. \tag{25}$$

Then we can get the expression of the aggregate price:

$$\widehat{P} = \frac{k\xi}{1-k(1-\xi)}(\widehat{Q}-\widehat{Y}^n). \tag{26}$$

Proof. See Appendix A4.

The asymmetric feature is kept in the price function of different sectors. The aggregate price is a function of nominal spending $\widehat{Q}$ and natural rate of aggregate output $\widehat{Y}^n$.

The relative price between the two sectors is expressed as

$$\widehat{P}_1 - \widehat{P}_2 = k(\widehat{A}_1 - \widehat{A}_2). \tag{27}$$

5 Optimal monetary policy

We now start to determine the optimal monetary policy. Equation (13) shows that the utility of the representative household relies on the output gap, the price dispersion in each sector and the deviation of the sectoral relative price from its natural value. The central bank responds to underlying productivity shocks as optimally as possible by affecting the nominal aggregate demand. On one hand, we know from (12) that monetary policy can affect the output gap. On the other hand, we know that monetary policy will affect the attention allocation strategy of firms in Section 3. The observational noise is proportional to the variance of the optimal price under full information which is influenced by the policy. So the monetary policy will also affect the price dispersion in each sector and the sectoral relative price.

In the following subsections we first analyze two extreme settings regarding information. Then we examine the optimal policy in the symmetric case where $\widehat{A}_1 = \widehat{A}_2$. At last we tackle the general case.

5.1 Two benchmarks

It is classical result that nominal neutrality holds under perfect information; see Lucas (1972). In this case, the optimal monetary policy stabilizes the output gap as well as the price level completely. However, the conclusion is not so obvious in the two-sector model in this paper. This subsection considers two extreme informational settings and proves that complete price stabilization and output stabilization is still optimal under perfect information.

5.1.1 *benchmark I: perfect information*

If firms have perfect information, we get the natural prices. Considering $p(i) = P_j$, the following relationship holds:

$$\widehat{P}^n = \widehat{Q} - \widehat{Y}_n, \tag{28}$$

$$\widehat{P}_1^n = \widehat{Q} - \widehat{Y}_n + n_2(\widehat{A}_1 - \widehat{A}_2), \tag{29}$$

$$\widehat{P}_2^n = \widehat{Q} - \widehat{Y}_n + n_1(\widehat{A}_2 - \widehat{A}_1). \tag{30}$$

The monetary neutrality holds again, in which case the output gap is zero. The price dispersion in each sector equals zero and the relative price of the two sectors is independent of the policy. It is optimal for the central bank to set $\widehat{Q} = \widehat{Y}_n$.

5.1.2 *benchmark II: no information*

When firms have no information processing capacity, it comes to the case of no information. In this case, firms do not respond to any shocks, which means all the shocks are treated as 'surprises', $\widehat{P} = \widehat{P}_1 = \widehat{P}_2 = 0$. Monetary policy has real effects only. $\widehat{Q} = \widehat{Y}_n$ is still optimal.

5.2 Policy in the symmetric case

If the two sectors share the same productivity shock, which is represented by$A_1 = A_2 = A$,[12] we say shocks are symmetric. In such case, we have:

$$\widehat{P}_j = \frac{k\xi}{1-k(1-\xi)}(\widehat{Q} - \widehat{Y}^n),$$
$$\widehat{P} = \frac{k\xi}{1-k(1-\xi)}(\widehat{Q} - \widehat{Y}^n).$$

The sectoral price and aggregate price are only functions of $\widehat{Q} - \widehat{Y}^n$. In this setting, the model is equivalent to the one-sector model in Adam(2007). In his paper, nominally accommodating productivity shocks is optimal because firms choose not to observe any information about the shocks in such case. So there is no price dispersion in each sector and no relative price distortion between the two sectors. The economy responds to the disturbances efficiently. The same logic applies here.

5.3 Policy with asymmetric shocks

The situation differs considerably when considering asymmetric shocks across sectors. For simplicity and tractability, in this paper we only consider independent shocks. A_1 and A_2 are assumed to be independent. Firms have limited attention summarized by

[12]Since A1 and A2 are random variables, here we mean A1 equals A2 in the sense of 'almost everywhere'.

k. Given the equilibrium of private sectors solved in section 4, the Ramsey problem of the central bank is given by:

$$\underset{\widehat{Q}}{\text{Max}} \quad -E\left\{\xi(\widehat{Y}-\widehat{Y}_n)^2+\alpha\textstyle\sum_{j=1}^{2}n_j^2\widehat{P}_j^2+\beta[(\widehat{P}_1-\widehat{P}_2)-(\widehat{P}_1^n-\widehat{P}_2^n)]^2\right\}$$

s.t.

$$\begin{aligned}
&\widehat{P}_1^n-\widehat{P}_2^n=\widehat{A}_1-\widehat{A}_2,\\
&\widehat{P}_1=\frac{k}{1-k(1-\xi)}\{\xi(\widehat{Q}-\widehat{Y}^n)+n_2[1-k(1-\xi)](\widehat{A}_1-\widehat{A}_2)\},\\
&\widehat{P}_2=\frac{k}{1-k(1-\xi)}\{\xi(\widehat{Q}-\widehat{Y}^n)+n_1[1-k(1-\xi)](\widehat{A}_2-\widehat{A}_1)\}.\\
&\widehat{Y}-\widehat{Y}^n=\frac{1-k}{1-k(1-\xi)}(\widehat{Q}-\widehat{Y}^n).
\end{aligned} \tag{31}$$

We find that the central bank can not influence the relative price distortions during the two sectors. If the monetary authority plans to completely stabilize the output gap, it should implement the following policy:

$$\widehat{Q}_y=\widehat{Y}^n. \tag{32}$$

However, this policy will not induce the optimal price dispersion and the optimal sectoral relative price. Actually, proposition 2 gives the optimal monetary policy explicitly, which is the solution of problem (31).

If the central bank tries to stabilize the price, i.e. minimize the aggregate price dispersion, which is the second term in the objective function, it should set the nominal spending to:

$$\widehat{Q}_p=\widehat{Y}^n+\frac{n_1n_2(n_2-n_1)[1-k(1-\xi)]}{(n_1^2+n_2^2)k\xi}(\widehat{A}_1-\widehat{A}_2). \tag{33}$$

Proposition 2 (Optimal monetary policy) *Consider the Ramsey problem (31), where the shocks A_1 and A_2 are assumed to be independent. The central bank controls nominal demand directly which is $\widehat{Q}=\widehat{P}+\widehat{Y}$. The solution to the Ramsey problem (31) is given by:*

$$\widehat{Q}=\widehat{Y}^n+B(\widehat{A}_1-\widehat{A}_2), \tag{34}$$

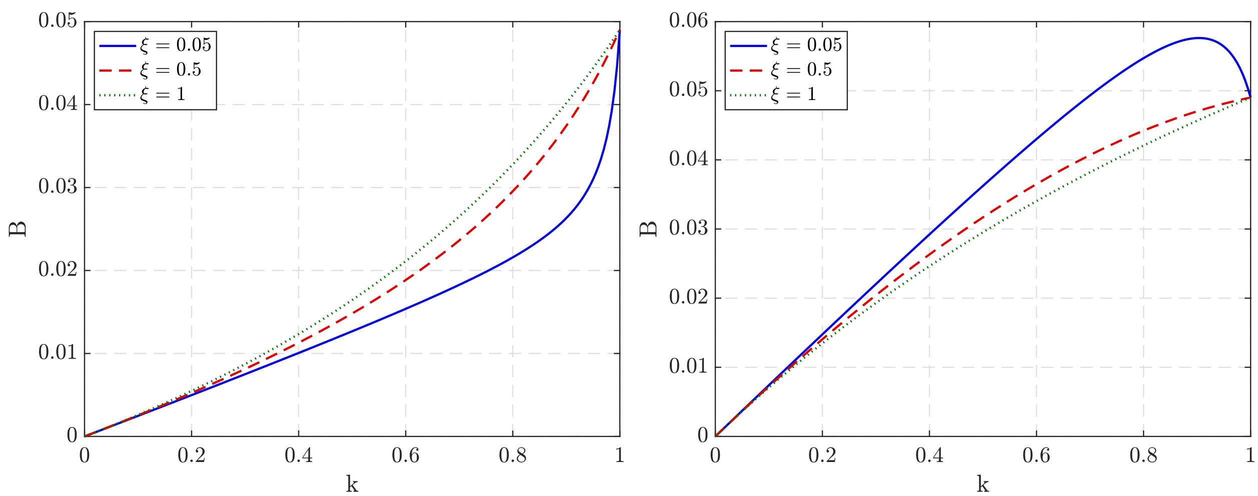
where

$$B = \frac{\theta n_1 n_2 (n_2 - n_1) k[1 - k(1-\xi)]}{2(1-k) + \theta(n_1^2 + n_2^2) k\xi}$$

which is the unique optimal monetary policy. where k is the information processing capacity, ξ represents the strategic complementarity across firms' pricing decisions. θ is the elasticity of substitution within sectors

Proof. The solution is readily calculated by maximizing the quadratic objective function and the uniqueness is also guaranteed by the form of quadratic function.

Figure 1: Weight on asymmetry of shocks



Note: In the left figure, the elasticity of substitution $\theta = 2$. In the right figure, the elasticity of substitution $\theta = 6$. $n_1 = 0.45$, $n_2 = 0.55$

The relationship between the weight on the asymmetry of shocks B and the information processing capacity and strategic complementarity is nonlinear, as shown in Figure 1, depending on the elasticity of substitution within sectors.

Compared with the symmetric case, there is a tradeoff between output and price stabilization. The optimal policy rule calculated above can be decomposed as

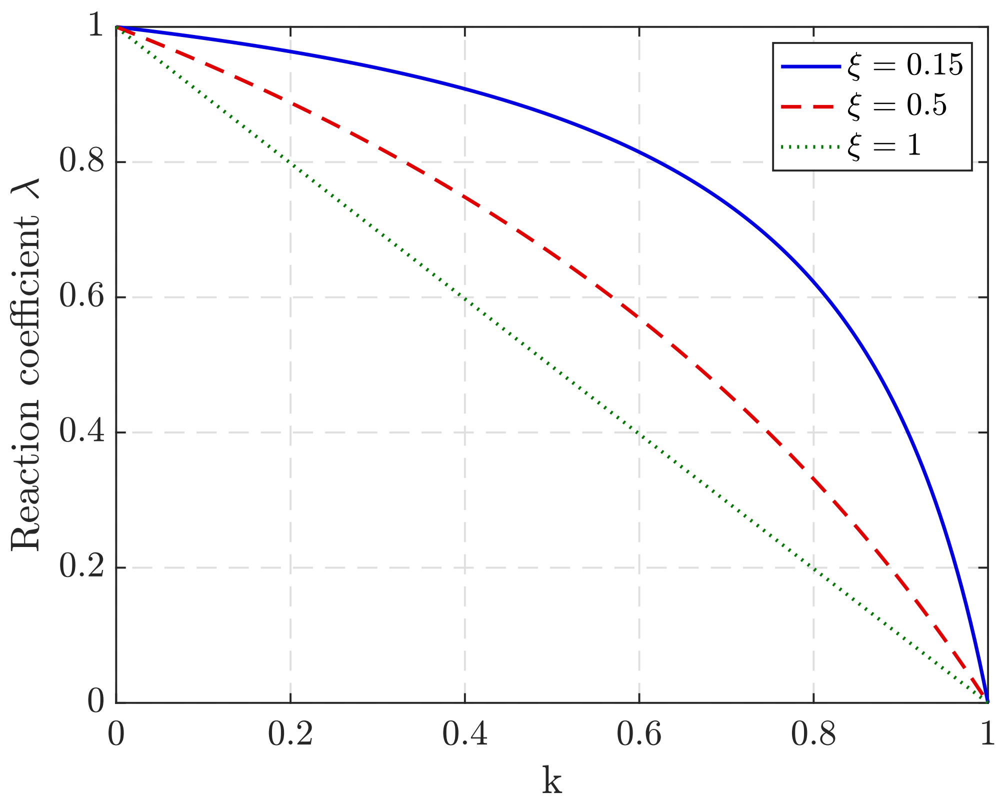
$$\widehat{Q} = \lambda \widehat{Q}_y + (1-\lambda)\widehat{Q}_p.$$

where $\widehat{Q}_y$ and $\widehat{Q}_p$ are the policies to solely stabilize output gap and price dispersion respectively. λ is the weight on output gap stabilization, which is:

$$\lambda = \frac{2(1-k)}{2(1-k) + \theta(n_1^2 + n_2^2)\xi k}$$

Taking second order derivative, we find that the reaction coefficient λ is both decreasing with information processing capacity k and the strategic complementarity ξ. Figure 2 illustrates how the optimal policy reaction coefficient β responds to different information processing capacity k varying from 0 to 1 for various degree of strategic complementarity. We find that given the degree of strategic complementarity, as the information processing capacity k increases, central bank should put more weight on price stabilization. Given information processing capacity k, the stronger the strategic complementarities, the more weight should be imposed on output gap stabilization.

Figure 2: Optimal policy reaction coefficient



Note: $\theta = 4$. $n_1 = 0.45$, $n_2 = 0.55$

As is shown above, the productivity shock in one sector exert an asymmetric influence on firms' price setting in that sector. Because the price dispersion in one sector is proportional to the observational noise of firms in that sector, different sectoral shocks affect the price dispersion in an asymmetric way. At the same time, according to the optimal information structure with limited attention, the observational noise of a firm is proportional to the variance of its optimal price under perfect information, which can be influenced by the monetary policy. The central bank can reduce the price dispersion by responding to the productivity shocks. However, due to the asymmetry of shocks, even if the central bank tries to stabilize the output gap, firms will still pay their attention to the shocks due to the asymmetry of shocks, which will cause price dispersion and relative price distortion. The asymmetric characteristics of the shocks with inattentive firms explain why price stabilization is no longer optimal. When responding to sectoral shocks, apart from stabilizing the output gap, the optimal monetary policy requires the central bank to put some weight to the asymmetry of these shocks. The coefficient B reflects the response of central bank to the asymmetry of shocks.

It is interesting to find a special case: when $n_1 = n_2$, the tradeoff between output gap stabilization and price stabilization disappears. This means, when the economy size is equal between the two sectors[13], output gap stabilization is optimal again. This result is also intuitive: when adding the variance of the sectoral prices together, the asymmetric effects of the two independent shocks to each sector cancel out if the economy size is same in the two sectors. The central bank can attain its multiple objective goals at the same time. But unlike the symmetric case, under the optimal policy, the allocation is not efficient, there are still price dispersions in each sector. This is because firms still pay positive attention to the shocks, and as a result, price dispersions induce distortions.

6 Conclusion

This paper solves an optimal monetary policy problem for a two-sector economy where monopolistic competitive firms pays limited attention to sectoral shocks. When the shocks are symmetric across sectors, complete price stabilization is optimal. However, when the shocks are asymmetric, there are tradeoffs between output and price stabilization. Complete price stabilization is no longer optimal, and the equivalence between output stabilization and price stabilization also breaks down. Central bank should coordinate conflictive welfare objectives by putting some weight to the asymmetry of sectoral shocks.

There are several possible extensions to this framework. One interesting topic is to allow different information processing capacity for different sectors. This work only models static case, so future work is needed to analyze the dynamic setting and the

[13] Aoki (2001) makes this assumption.

corresponding optimal policy.[14] Another interesting topic is to consider a model with sectoral shocks together with aggregate and firm specific idiosyncratic shocks and analyze their interactions. This paper only considers exogenous information processing capacity. It is interesting to check whether the conclusion still holds in a setting with endogenous attention choice.[15] Finally, to generalize this framework to investigate the optimal monetary policy of a monetary union is another important topic.

[14]Although it is technically challenging, potentially we can use the method proposed by Huo and Takayama (2015) to solve the rational expectation with higher order beliefs.

[15]We are grateful to Klaus Adam and Mirko Wiederholt for pointing this to us.

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Appendix

A Proofs and Derivations

A.1 Pricesetting function of firms

All the log-linear and log-quadratic approximation below is expanded around a deterministic steady state with $y(i) = Y_j = Y = \overline{Y}, p(i) = P_j = P = \overline{P}, A_j = \overline{A} = 1$. In the steady state, allocation is efficient which satisfies

$$\frac{V'(\overline{Y})}{U'(\overline{Y})} = 1$$

Under the assumption specified in Section 2, first-order condition of profit maximization of firm i is

$$p(i) = E[Pm_j(y(i), Y; A_1, A_2)|s_i], \tag{35}$$

where m_j is the real marginal cost of supplying good i in sector j given by

$$m_j(y(i), Y; A_1, A_2) = A_j \frac{V'(L)}{U'(Y)}, \tag{36}$$

where

$$L = \int_{N_1} A_1 y(i)\, \mathrm{d}i + \int_{N_2} A_2 y(i)\, \mathrm{d}i.$$

After log-linear approximation, it is shown that

$$\widehat{L} = \widehat{Y} + n_1 \widehat{A}_1 + n_2 \widehat{A}_2.$$

We can then define the natural rate of output Y_j for each sector j as the common equilibrium output of each good i in that sector under perfect information with flexible prices. It is implicitly defined as

$$(1+\tau)\mu m_j(Y_j^n, Y^n; A_1, A_2) = \left(\frac{Y_j^n}{n_j Y^n}\right)^{-\frac{1}{\eta}} \tag{37}$$

Here μ is the desired markup defined as $\mu = \frac{\theta}{\theta-1}$. Y^n is given by substituting Y_j^n into (1).

Given the expressions above, log-linearizing (36) around the steady state delivers

$$\begin{aligned}\widehat{m}_j\big(y(i), Y; A_1, A_2\big) &= \overline{Y}\frac{V''(\overline{Y})}{U'(\overline{Y})}\widehat{L} - \overline{Y}\frac{U''(\bar{(}Y)V'(\overline{Y})}{U'^2(\overline{Y})}\widehat{Y} + \widehat{A}_j \\ &= \xi\widehat{Y} + \omega n_1\widehat{A}_1 + \omega n_2\widehat{A}_2 + \widehat{A}_j\end{aligned} \tag{38}$$

We get the expression of natural rate of aggregate output $\widehat{Y}^n$ after log-linearizing (37) together with (38)

$$\widehat{Y}^n = -(\omega n_1\widehat{A}_1 + \omega n_2\widehat{A}_2) - (n_1\widehat{A}_1 + n_2\widehat{A}_2). \tag{39}$$

Substitute this equation to (38), we have

$$\widehat{m}_j\big(y(i), Y; A_1, A_2\big) = \xi(\widehat{Y} - \widehat{Y}_n) - (n_1\widehat{A}_1 + n_2\widehat{A}_2) + \widehat{A}_j \tag{40}$$

Hence by log-linearizing (35) and substituting (40) into it we can get (10)

A.2 Objective function of central bank

Consider the utility function specified in Section 2. The second-order expansion of the first term of the utility of the household can be derived as

$$\begin{aligned}U(Y) &= U'(\overline{Y})(Y - \overline{Y}) + \tfrac{1}{2}U''(\overline{Y})(Y - \overline{Y})^2 + o(2) \\ &= \overline{Y}U'(\overline{Y})\widehat{Y} + \tfrac{1}{2}\Big(U'(\overline{Y})\overline{Y} + U''(\overline{Y})\overline{Y}^2\Big)\widehat{Y}^2 + o(2)\end{aligned}$$

The second equality follows from the taylor series expansion

$$Y/\overline{Y} = 1 + \widehat{Y} + \tfrac{1}{2}\widehat{Y}^2 + o(2).$$

We can express L as

$$L = A_1\widetilde{Y}_1 + A_2\widetilde{Y}_2 \tag{41}$$

where $\widetilde{Y}_j = \int_{n_j} y(i)\, \mathrm{d}i$ represents the output of that sector.

Then the second term can be approximated around the steady state to second order expressed as

$$\begin{aligned}V(L) &= V(A_1\widetilde{Y}_1 + A_2\widetilde{Y}_2) \\ &= V'(\overline{Y})(\widetilde{Y}_1 - n_1\overline{Y}) + V'(\overline{Y})(\widetilde{Y}_2 - n_2\overline{Y})\end{aligned}$$

$$
\begin{aligned}
&+\tfrac{1}{2}V''(\overline{Y})(\widetilde{Y}_1-n_1\overline{Y})^2+\tfrac{1}{2}V''(\overline{Y})(\widetilde{Y}_2-n_2\overline{Y})^2\\
&+(V'(\overline{Y})+n_1\overline{Y}V''(\overline{Y}))(\widetilde{Y}_1-n_1\overline{Y})(A_1-1)+n_1\overline{Y}V''(\overline{Y})(\widetilde{Y}_2-n_2\overline{Y})(A_1-1)\\
&+n_2\overline{Y}V''(\overline{Y})(\widetilde{Y}_1-n_1\overline{Y})(A_2-1)+(V'(\overline{Y})+n_2\overline{Y}V''(\overline{Y}))(\widetilde{Y}_2-n_2\overline{Y})(A_2-1)\\
&+V''(\overline{Y})(\widetilde{Y}_1-n_1\overline{Y})(\widetilde{Y}_2-n_2\overline{Y})+\text{t.i.p}+o(2)\\
=\;&\overline{Y}V'(\overline{Y})(n_1\widehat{\widetilde{Y}}_1+n_2\widehat{\widetilde{Y}}_2+\tfrac{1}{2}n_1\widehat{\widetilde{Y}}_1^2+\tfrac{1}{2}n_2\widehat{\widetilde{Y}}_2^2+\tfrac{1}{2}n_1^2\omega\widehat{\widetilde{Y}}_1^2+\tfrac{1}{2}n_2^2\omega\widehat{\widetilde{Y}}_2^2+n_1n_2\omega\widehat{\widetilde{Y}}_1\widehat{\widetilde{Y}}_2\\
&+n_1\widehat{\widetilde{Y}}_1\widehat{A}_1+n_1\widehat{\widetilde{Y}}_2\widehat{A}_2+n_1^2\omega\widehat{\widetilde{Y}}_1\widehat{A}_1+n_1n_2\omega\widehat{\widetilde{Y}}_2\widehat{A}_1+n_1n_2\omega\widehat{\widetilde{Y}}_1\widehat{A}_2+n_2^2\omega\widehat{\widetilde{Y}}_2\widehat{A}_2)\\
&+\text{t.i.p}+o(2)\\
=\;&\overline{Y}V'(\overline{Y})(n_1\widehat{Y}_1+n_2\widehat{Y}_2+\tfrac{1}{2}n_1\widehat{Y}_1^2+\tfrac{1}{2}n_2\widehat{Y}_2^2+\tfrac{1}{2}n_1^2\omega\widehat{Y}_1^2+\tfrac{1}{2}n_2^2\omega\widehat{Y}_2^2\\
&+n_1\widehat{Y}_1\widehat{A}_1+n_2\widehat{Y}_2\widehat{A}_2+n_1^2\omega\widehat{Y}_1\widehat{A}_1+n_1n_2\omega\widehat{Y}_2\widehat{A}_1+n_1n_2\omega\widehat{Y}_1\widehat{A}_2\\
&+n_2^2\omega\widehat{Y}_2\widehat{A}_2+n_1n_2\omega\widehat{Y}_1\widehat{Y}_2+\tfrac{1}{2\theta}\textstyle\sum_{j=1}^2 n_j var_i^j y(i))+\text{t.i.p}+o(2).\\
=\;&\overline{Y}V'(\overline{Y})[\widehat{Y}+\tfrac{1}{2}(1+\omega)\widehat{Y}^2+\omega(n_1\widehat{A}_1+n_2\widehat{A}_2)\widehat{Y}+n_1\widehat{A}_1\widehat{Y}_1+n_2\widehat{A}_2\widehat{Y}_2\\
&+\tfrac{1}{2}n_1n_2\eta^{-1}(\widehat{Y}_1-\widehat{Y}_1)^2+\tfrac{1}{2\theta}\textstyle\sum_{j=1}^2 n_j var_i^j y(i)]+\text{t.i.p}+o(2)\\
=\;&\overline{Y}V'(\overline{Y})[\widehat{Y}+\tfrac{1}{2}(1+\omega)\widehat{Y}^2-\xi\widehat{Y}\widehat{Y}^n+\tfrac{1}{2}n_1n_2\eta^{-1}[(\widehat{Y}_1-\widehat{Y}_1)-(\widehat{Y}_1^n-\widehat{Y}_2^n)]^2\\
&+\tfrac{1}{2\theta}\textstyle\sum_{j=1}^2 n_j var_i^j y(i)]+\text{t.i.p}+o(2)
\end{aligned}
$$

The fourth equality follows the second-order Taylor expansion of $\widetilde{Y}_j=\int_{n_j}y(i)\,\mathrm{d}i$ given by

$$
\widehat{\widetilde{Y}}_j=\widehat{Y}_j+\tfrac{1}{2\theta}var_i^j y(i)+o(2), \tag{42}
$$

and the fifth equality makes use of the two equations below

$$
n_1\widehat{Y}_1+n_2\widehat{Y}_2=\widehat{Y}-\tfrac{n_1n_2(1-\eta^{-1})}{2}(\widehat{Y}_1-\widehat{Y}_2)^2,
$$

$$
n_1\widehat{Y}_1^2+n_2\widehat{Y}_2^2=n_1n_2(\widehat{Y}_1-\widehat{Y}_2)^2+\widehat{Y}^2.
$$

We get the sixth equality by using equation (39) and the fact that:

$$
\eta^{-1}(\widehat{Y}_1^n-\widehat{Y}_2^n)=\widehat{A}_2-\widehat{A}_1
$$

and we also use the following factoring techniques:

$$
\begin{aligned}
n_1\widehat{A}_1\widehat{Y}_1 + n_2\widehat{A}_2\widehat{Y}_2 &= n_1(n_1+n_2)\widehat{Y}_1\widehat{A}_1 + n_1n_2\widehat{Y}_1\widehat{A}_2 - n_1n_2\widehat{Y}_1\widehat{A}_2 \\
&+ n_2(n_1+n_2)\widehat{Y}_1\widehat{A}_1 + n_1n_2\widehat{Y}_2\widehat{A}_1 - n_1n_2\widehat{Y}_2\widehat{A}_1 \\
&= \xi\widehat{Y}\widehat{Y}^n - n_1n_2\eta^{-1}(\widehat{Y}_1 - \widehat{Y}_2)(\widehat{Y}_1^n - \widehat{Y}_2^n).
\end{aligned}
$$

Then the utility of the representative household is approximated by

$$
\begin{aligned}
\Omega &= U(Y) - V(L) \\
&= -\tfrac{1}{2}\overline{Y}U'[\xi\widehat{Y}^2 - 2\xi\widehat{Y}_n\widehat{Y} + n_1n_2\eta^{-1}[(\widehat{Y}_1 - \widehat{Y}_2) - (\widehat{Y}_1^n - \widehat{Y}_2^n)]^2 \\
&\quad + \tfrac{1}{2\theta}\textstyle\sum_{j=1}^2 n_j var_i^j y(i)] + \text{t.i.p} + o(2) \\
&= -\tfrac{1}{2}\overline{Y}U'[\xi(\widehat{Y} - \widehat{Y}_n)^2 + n_1n_2\eta[(\widehat{P}_1 - \widehat{P}_2) - (\widehat{P}_1^n - \widehat{P}_2^n)]^2 \\
&\quad + \tfrac{1}{2\theta}\textstyle\sum_{j=1}^2 n_j var_i^j y(i)]\text{t.i.p} + o(2).
\end{aligned}
$$

Here the third equality uses the fact that

$$\widehat{Y}_j = \widehat{Y} - \eta(\widehat{P}_j - \widehat{P})$$

which comes from log-linearization of (5) around the steady state specified above. Taking the relationship between $var_i^j y(i)$ and $var_i^j p(i)$ given by

$$var_i^j y(i) = \theta^2 var_i^j p(i)$$

into consideration, we finally get the expansion of utility of the representative household expressed as

$$
\begin{aligned}
\Omega &= U(Y) - V(L) \\
&= -\tfrac{1}{2}\overline{Y}U'\left\{\xi(\widehat{Y} - \widehat{Y}_n)^2 + \alpha\textstyle\sum_{j=1}^2 n_j^2\widehat{P}_j^{\,2} + \beta[(\widehat{P}_1 - \widehat{P}_2) - (\widehat{P}_1^n - \widehat{P}_2^n)]^2\right\} + \text{t.i.p} + o(2).
\end{aligned}
$$

where $\alpha = \frac{(1-k)\theta}{2k}$ and $\beta = n_1n_2\eta$.

A.3 Profit loss of firms

We expand the profit loss $\Pi[\hat{p}^{\circ}(i),\cdot] - \Pi[\hat{p}^{*}(i),\cdot]$ of firm i up to second order:

$$\begin{aligned}
\widehat{\Pi}[\hat{p}^{\circ}(i),\cdot] - \widehat{\Pi}[\hat{p}^{*}(i),\cdot] &= -\tfrac{\theta n_j \overline{Y}}{2\overline{P}}[\hat{p}^{\circ}(i)]^2 + \tfrac{\theta n_j \overline{Y}}{2\overline{P}}[\hat{p}^{*}(i)]^2 \\
&\quad + \tfrac{\theta n_j \overline{Y}}{2\overline{P}}[\widehat{P} + \xi\widehat{Y} + (\omega n_1 \widehat{A}_1 + \omega n_2 \widehat{A}_2) + \widehat{A}_j][\hat{p}^{\circ}(i) - \hat{p}^{*}(i)] + o(2) \\
&= \tfrac{\theta n_j \overline{Y}}{2\overline{P}}[\hat{p}^{\circ}(i) - \hat{p}^{*}(i)]^2 + o(2).
\end{aligned}$$

Hence we have got equation (17).

A.4 Optimal information Structure

Give signal s_i, the optimal price is $\hat{p}^{*}(i) = \mathbb{E}[\hat{p}^{\circ}(i)|s_i]$, so that the objective function for the information processing problem is $\mathrm{Var}(\hat{p}^{\circ}(i)|s_i)$ I refer reader to Adam (2007) Appendix A.3 for the rest of the proof, since it is almost identical for the rest of the proof.

A.5 Equilibrium

Method of undetermined coefficients is used to solve the Rational Expectation Equilibrium. We suppose the price functions of each sector are $\widehat{P}_j = a_j\widehat{A}_1 + b_j\widehat{A}_2$ respectively, and the monetary policy is $\widehat{Q} = g_1\widehat{A}_1 + g_2\widehat{A}_2$. Substitute these expressions to (23) we have:

$$\begin{aligned}
a_1\widehat{A}_1 + b_1\widehat{A}_2 &= k\{\xi(g_1\widehat{A}_1 + g_2\widehat{A}_2) + (1-\xi)[n_1(a_1\widehat{A}_1 + b_1\widehat{A}_2) + n_2(a_2\widehat{A}_1 + b_2\widehat{A}_2)] \\
&\quad + \omega n_1\widehat{A}_1 + \omega n_2\widehat{A}_2 + \widehat{A}_1\} \\
a_2\widehat{A}_1 + b_2\widehat{A}_2 &= k\{\xi(g_1\widehat{A}_1 + g_2\widehat{A}_2) + (1-\xi)[n_1(a_1\widehat{A}_1 + b_1\widehat{A}_2) + n_2(a_2\widehat{A}_1 + b_2\widehat{A}_2)] \\
&\quad + \omega n_1\widehat{A}_1 + \omega n_2\widehat{A}_2 + \widehat{A}_2\}
\end{aligned}$$

Equalizing the corresponding undetermined coefficients delivers the solution given by

$$\begin{cases} a_1 = \dfrac{k}{1-k(1-\xi)}[\xi g_1 + \omega n_1 + 1 - n_2 k(1-\xi)] \\ b_1 = \dfrac{k}{1-k(1-\xi)}[\xi g_2 + \omega n_2 + n_2 k(1-\xi)] \\ a_2 = \dfrac{k}{1-k(1-\xi)}[\xi g_1 + \omega n_1 + n_1 k(1-\xi)] \\ b_2 = \dfrac{k}{1-k(1-\xi)}[\xi g_2 + \omega n_2 + 1 - n_1 k(1-\xi)] \end{cases}$$

Here we get the expressions of (24) and (25).